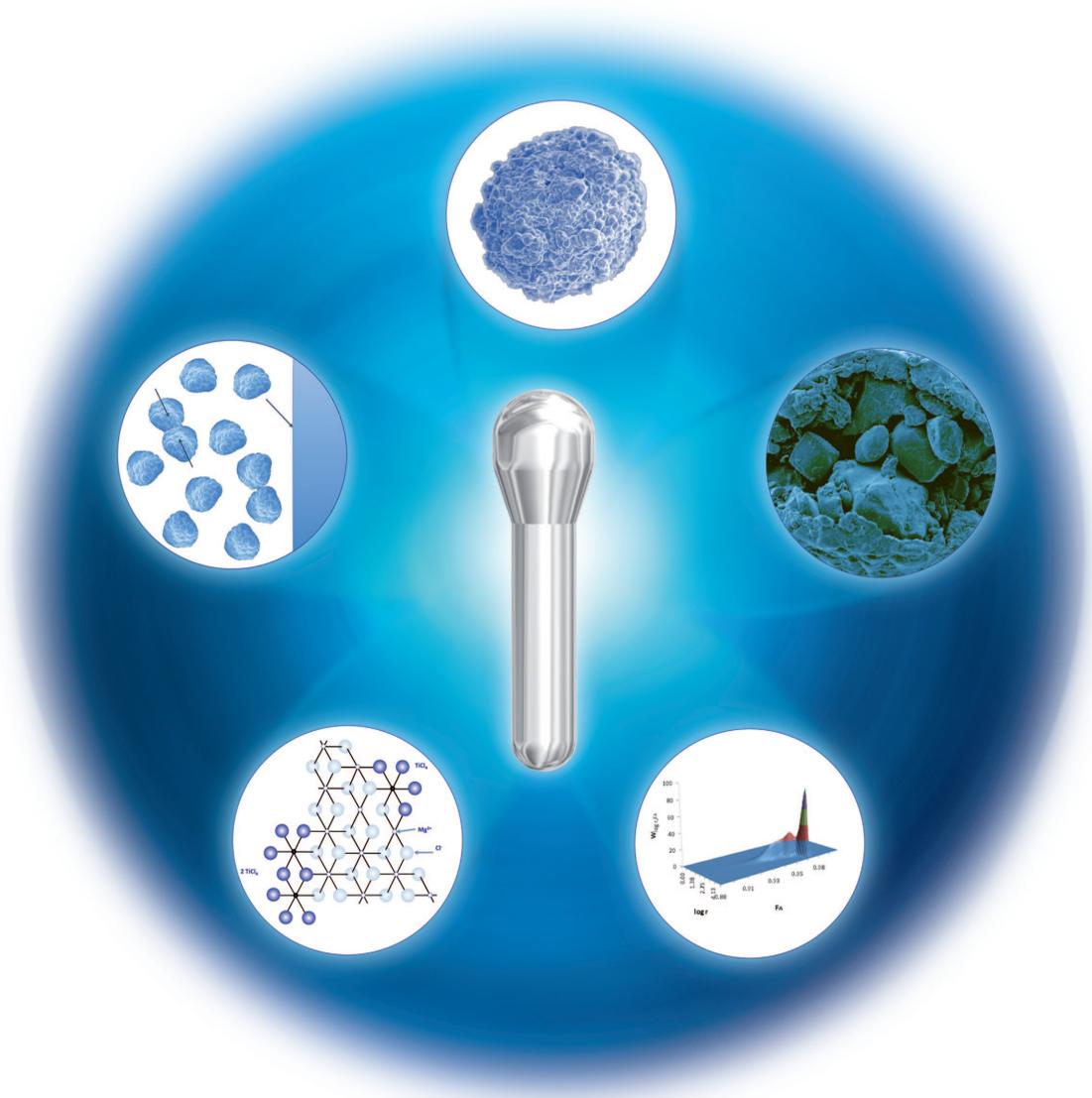


J. B. P. Soares, T. F. L. McKenna

 WILEY-VCH

# Polyolefin Reaction Engineering



## Contents

**Acknowledgments** *XI*

**Preface** *XIII*

**Nomenclature** *XVII*

<b>1</b>	<b>Introduction to Polyolefins</b>	<b>1</b>
1.1	Introduction	1
1.2	Polyethylene Resins	4
1.3	Polypropylene Resins	10
	Further Reading	13
<b>2</b>	<b>Polyolefin Microstructural Characterization</b>	<b>15</b>
2.1	Introduction	15
2.2	Molecular Weight Distribution	17
2.2.1	Size Exclusion Chromatography	17
2.2.2	Field Flow Fractionation	27
2.3	Chemical Composition Distribution	29
2.3.1	Crystallizability-Based Techniques	29
2.3.2	High-Performance Liquid Chromatography	40
2.4	Cross-Fractionation Techniques	43
2.5	Long-Chain Branching	46
	Further Reading	51
<b>3</b>	<b>Polymerization Catalysis and Mechanism</b>	<b>53</b>
3.1	Introduction	53
3.2	Catalyst Types	56
3.2.1	Ziegler–Natta Catalysts	56
3.2.2	Phillips Catalysts	61
3.2.3	Metallocenes	62
3.2.4	Late Transition Metal Catalysts	67
3.3	Supporting Single-Site Catalysts	70

3.4	Polymerization Mechanism with Coordination Catalysts	76
	Further Reading	86
<b>4</b>	<b>Polyolefin Reactors and Processes</b>	<b>87</b>
4.1	Introduction	87
4.2	Reactor Configurations and Design	89
4.2.1	Gas-Phase Reactors	90
4.2.1.1	Fluidized Bed Gas-Phase Reactors	91
4.2.1.2	Vertical Stirred Bed Reactor	97
4.2.1.3	Horizontal Stirred Gas-Phase Reactor	99
4.2.1.4	Multizone Circulating Reactor	102
4.2.2	Slurry-Phase Reactors	104
4.2.2.1	Autoclaves	105
4.2.2.2	Slurry Loop Reactors	106
4.2.3	Solution Reactors	107
4.2.4	Summary of Reactor Types for Olefin Polymerization	108
4.3	Olefin Polymerization Processes	109
4.3.1	Polyethylene Manufacturing Processes	112
4.3.1.1	Slurry (Inert Diluent) Processes	112
4.3.1.2	Gas-Phase Processes	115
4.3.1.3	Mixed-Phase Processes	118
4.3.1.4	Solution Processes	119
4.3.2	Polypropylene Manufacturing Processes	121
4.3.2.1	Slurry (Inert Diluent) Processes	122
4.3.2.2	Gas-Phase Processes	122
4.3.2.3	Mixed-Phase Processes	125
4.4	Conclusion	128
	References	128
	Further Reading	129
<b>5</b>	<b>Polymerization Kinetics</b>	<b>131</b>
5.1	Introduction	131
5.2	Fundamental Model for Polymerization Kinetics	134
5.2.1	Single-Site Catalysts	134
5.2.1.1	Homopolymerization	134
5.2.1.2	Copolymerization	145
5.2.2	Multiple-Site Catalysts	149
5.2.3	Temperature Dependence of Kinetic Constants	152
5.2.4	Number of Moles of Active Sites	154
5.3	Nonstandard Polymerization Kinetics Models	156
5.3.1	Polymerization Orders Greater than One	156
5.3.2	Hydrogen Effect on the Polymerization Rate	161
5.3.3	Comonomer Effect on the Polymerization Rate	173
5.3.4	Negative Polymerization Orders with Late Transition Metal Catalysts	179

5.4	Vapor-Liquid-Solid Equilibrium Considerations	181
	Further Reading	184
<b>6</b>	<b>Polyolefin Microstructural Modeling</b>	<b>187</b>
6.1	Introduction	187
6.2	Instantaneous Distributions	188
6.2.1	Molecular Weight Distribution	188
6.2.1.1	Single-Site Catalysts	188
6.2.1.2	Multiple-Site Catalysts	199
6.2.2	Chemical Composition Distribution	212
6.2.2.1	Single-Site Catalysts	212
6.2.2.2	Multiple-Site Catalysts	222
6.2.3	Comonomer Sequence Length Distribution	232
6.2.4	Long-Chain Branching Distribution	237
6.2.5	Polypropylene: Regio- and Stereoregularity	250
6.3	Monte Carlo Simulation	251
6.3.1	Steady-State Monte Carlo Models	252
6.3.2	Dynamic Monte Carlo Models	262
	Further Reading	268
<b>7</b>	<b>Particle Growth and Single Particle Modeling</b>	<b>271</b>
7.1	Introduction	271
7.2	Particle Fragmentation and Growth	274
7.2.1	The Fragmentation Step	275
7.2.2	Particle Growth	284
7.3	Single Particle Models	286
7.3.1	Particle Mass and Energy Balances: the Multigrain Model (MGM)	287
7.3.2	The Polymer Flow Model (PFM)	292
7.3.3	An Analysis of Particle Growth with the MGM/PFM Approach	295
7.3.4	Convection in the Particles – High Mass Transfer Rates at Short Times	301
7.4	Limitations of the PFM/MGM Approach: Particle Morphology	304
	References	307
	Further Reading	307
<b>8</b>	<b>Developing Models for Industrial Reactors</b>	<b>311</b>
8.1	Introduction	311
	References	321
	Further Reading	322
	<b>Index</b>	<b>325</b>

## Preface

It is the mark of an instructed mind to rest satisfied with the degree of precision which the nature of the subject permits and not to seek exactness where only an approximation of the truth is possible.

*Aristotle (384–322 BC)*

The art of being wise is the art of knowing what to overlook.

*William James (1842–1910)*

The manufacture of polyolefins with coordination catalysts has been a leading force in the synthetic plastic industry since the early 1960s. Owing to the constant developments in catalysis, polymerization processes, and polyolefin characterization instruments, it continues to be a vibrant area of research and development today.

We have been working in this area for over 15 years, always feeling that there was a need for a book that summarized the most important aspects of polyolefin reaction engineering. This book reflects our views on this important industry. It grew out of interactions with the polyolefin industry through consulting activities and short courses, where we first detected a clear need to summarize, in one single source, the most generally accepted theories in olefin polymerization kinetics, catalysis, particle growth, and polyolefin characterization.

As quoted from Aristotle above, we will *rest satisfied with the degree of precision which the nature of the subject permits* and hope that our readers agree with us that this is indeed *the mark of an instructed mind*. It was not our intention to perform an extensive scholarly review of the literature for each of the topics covered in this book. We felt that this approach would lead to a long and tedious text that would become quickly outdated; several excellent reviews summarizing the most recent findings on polyolefin manufacturing and characterization are published regularly and are more adequate for this purpose. Instead, we present our interpretation of the field of polyolefin reaction engineering. Since any selection process is always subjective, we may have left out some approaches considered to be relevant by others, but we tried to be as encompassing as possible, considering the limitations of a book of this type. We have also sparsely used references in the main body of the chapters but added reference sections at their end where we discussed some

alternative theories, presented exceptions to the general approach followed in the chapters, and suggested additional readings. The reference sections are not meant to be exhaustive compilations of the literature but sources of supplemental readings and a door to the vast literature in the area. We hope this approach will make this book a pleasant reading and also provide the reader with additional sources of reference.

Chapter 1 introduces the field of polyolefins, with an overview on polyolefin types, catalyst systems, and reactor configurations. We also introduce our general philosophy of using mathematical models to link polymerization kinetics, mass and heat transfer processes at several length scales, and polymer microstructure characterization for a complete understanding of olefin polymerization processes.

We discuss polyolefin microstructure, as defined by their distributions of molecular weight, chemical composition, stereo- and regioregularity, and long-chain branching, in Chapter 2. It is not an overstatement to say that among all synthetic polymers, polyolefins are the ones where microstructure control is the most important concern. Polyolefin microstructure is a constant theme in all chapters of this book and is our best guide to understanding catalysis, kinetics, mass and heat transfer resistances, and reactor behavior.

Chapter 3 is dedicated to polymerization catalysis and mechanisms. The field of coordination catalysis is huge and, undoubtedly, the main driving force behind innovation in the polyolefin manufacturing industry; to give it proper treatment, a separate book would be necessary. Rather, we decided to focus on the most salient aspects of the several classes of olefin catalysts, their general behavior patterns and mechanisms, and how they can be related to polymerization kinetics and polyolefin microstructural properties.

The subject of Chapter 4, polymerization reactors, is particularly dear to us, polymer reactor engineers. In fact, polyolefin manufacturing is a “dream come true” for polymer reactor engineers because practically all possible configurations of chemical reactors can be encountered. A great deal of creativity went into reactor design, heat removal strategies, series and parallel reactor arrangements, and cost reduction schemes of polyolefin reactors. We start the chapter by discussing reactor configurations used in olefin polymerization and then continue with a description of the leading processes for polyethylene and polypropylene production.

Chapter 5 is the first chapter dedicated to the mathematical modeling of olefin polymerization. We start our derivations with what we like to call the fundamental model for olefin polymerization kinetics and develop, from basic principles, its most general expressions for the rates of catalyst activation, polymerization, and catalyst deactivation. The fundamental model, albeit widely used, does not account for several phenomena encountered in olefin polymerization; therefore, some alternative polymerization kinetic schemes are discussed at the end of this chapter.

In Chapter 6, we develop mathematical models to describe the microstructure of polyolefins. This is one of the core chapters of the book and helps connect polymerization kinetics, catalysis, and mass and heat transfer resistances to final polymer performance. We opted to keep the mathematical treatment as simple

as possible, without compromising the most relevant aspects of this important subject.

Particle fragmentation and growth are covered in Chapter 7. These models are collectively called *single particle models* and can be subdivided into polymer growth models and morphology development models. The two most well-established particle growth models are the polymeric flow model and the multigrain model. These models are used to describe heat and mass transfer in the polymeric particle after fragmentation takes place. The fragmentation of the catalyst particles themselves (described with morphology development models) is much harder to model, and there is still no well-accepted quantitative model to tackle this important subject. We review the main modeling alternatives in this field.

Finally, Chapter 8 is dedicated to macroscopic reactor modeling. This chapter is, in a way, the most conventional chapter from the chemical engineering point of view, since it involves well-known concepts of reactor residence time distribution, micromixing and macromixing, and reactor heat removal issues. The combination of macroscopic reactor models, single particle models, detailed polymerization kinetics, and polymer microstructural distributions, however, is very challenging and represents the ultimate goal of polyolefin reactor engineers.

## Nomenclature

What's in a name? William Shakespeare (1564–1616)

### Acronyms

CCD	chemical composition distribution
CEF	crystallization elution fractionation
CFC	cross-fractionation
CGC	constrained geometry catalyst
CLD	chain length distribution
CRYSTAF	crystallization analysis fractionation
CSLD	comonomer sequence length distribution
CSTR	continuous stirred tank reactor
CXRT	computed X-ray tomography
DEAC	diethyl aluminum chloride
DIBP	di-iso-butylphthalate
DSC	differential scanning calorimetry
EAO	ethylaluminumoxane
EB	ethyl benzoate
EDX	energy dispersive X-ray spectroscopy
EGMBE	ethylene glycol monobutylether
ELSD	evaporative light scattering detector
EPDM	ethylene-propylene-diene monomer rubber
EPR	ethylene-propylene rubber
FBR	fluidized bed reactor
FFF	field flow fractionation
FTIR	Fourier-transform infrared
GPC	gel permeation chromatography
HDPE	high-density polyethylene
HMDS	hexamethyldisilazine
HPLC	high-performance liquid chromatography
HSBR	horizontal stirred bed reactor

IR	infrared
LALLS	low-angle laser light scattering
LCB	long-chain branch
LDPE	low-density polyethylene
LLDPE	linear low-density polyethylene
LS	light scattering
MALLS	multiangle laser light scattering
MAO	methylaluminoxane
MDPE	medium-density polyethylene
MFI	melt flow index
MFR	melt flow rate
MGM	multigrain model
MI	melt index
MWD	molecular weight distribution
MZCR	multizone circulating reactor
NMR	nuclear magnetic resonance
NPTMS	<i>n</i> -propyltrimethoxysilane
ODCB	orthodichlorobenzene
PDI	polydispersity index
PFM	polymer flow model
PFR	plug flow reactor
PP	polypropylene
PSD	particle size distribution
RND	random number generated in the interval [0,1]
RTD	residence time distribution
SCB	short-chain branch
SEC	size exclusion chromatography
SEM	scanning electron microscopy
SLD	sequence length distribution
SPM	single particle model
<i>t</i> BAO	<i>t</i> -butylaluminoxane
TCB	trichlorobenzene
TEA	triethyl aluminum
TEM	transmission electron microscopy
TGIC	temperature gradient interaction chromatography
TMA	trimethyl aluminum
TOF	turnover frequency
TREF	temperature rising elution fractionation
UHMWPE	ultrahigh-molecular weight polyethylene
ULDPE	ultralow-density polyethylene
VLDPE	very low-density polyethylene
VISC	viscometer
VSBR	vertical stirred bed reactor

## Symbols

$a$	Mark–Houwink equation constant, Eq. (2.7)
$a_s$	specific surface area of the support
$A$	monomer type A
$A$	total reactor heat transfer area
$A_i$	Arrhenius law preexponential factor for reaction of type $i$
$A_S$	support specific surface area
$Al$	cocatalysts
$[AS^*]$	concentration of active sites per unit surface area in the microparticle
$B$	monomer type B
$B_n$	average number of long-chain branches per polymer chain
$C$	catalyst precursor or active site
$C^*$	active site
$[C_0]$	initial concentration of active sites
$C_d$	deactivated catalytic site
$C_p$	heat capacity
$D_b$	bulk diffusivity
$D_{eff}$	effective diffusivity in the macroparticle
$d_p$	polymer (or catalyst) particle diameter
$D_p$	diffusivity in the primary particle
$D_r$	dead polymer chain
$D_{r,i}$	dead polymer chain of length $r$ having $i$ long-chain branches
$\overline{D}_{r,i}$	dead polymer chain of length $r$ having $i$ long-chain branches and a terminal unsaturation (macromonomer)
$E(t)$	reactor residence time distribution
$E_i$	Arrhenius law activation energy for reaction of type $i$
$f^=$	molar fraction of macromonomers in the reactor
$f_i$	molar fraction of monomer type $i$ in the polymerization medium
$\overline{f}_r$	frequency Flory chain length distribution, Eq. (6.13)
$\overline{\overline{f}}_r$	overall frequency chain length distribution for chain having long-chain branches, Eq. (6.101)
$f_{rk}$	frequency chain length distribution for chains with $k$ long-chain branches per chain, Eq. (6.86)
$f_{\log r k}$	frequency chain length distribution for chains with $k$ long-chain branches per chain, log scale, Eq. (6.88)
$F$	monomer molar flow rate to the reactor
$F_A$	comonomer molar fraction in the copolymer
$\overline{F}_A$	average comonomer molar fraction in the copolymer
$F_{Br}$	molar fraction of comonomer B as a function of chain length
$F_{M,in}$	molar flow rate of the monomer feed to the reactor
$F_{M,out}$	molar flow rate of the monomer exiting the reactor
$g$	branching index, Eq. (2.18)
$g'$	viscosity branching index, Eq. (2.17)
$\Delta G$	Gibbs free energy change

$h$	average convective heat transfer coefficient between the macroparticle and surroundings
$\Delta H$	enthalpy change
$\Delta H_p$	average enthalpy of polymerization
$\Delta H_r$	enthalpy of reaction
$\Delta H_u$	enthalpy of melting for a crystallizable repeating unit, Eq. (2.26)
$\Delta H_{\text{vap}}$	enthalpy of vaporization
$I_1$	Bessel function of the first kind and order 1
$k_a$	site activation rate constant
$k_c, k_c^-$	forward and reverse rate constants, respectively, for the formation of dormant site with Ni-diimine catalysts, Table 5.8
$k_d$	first-order deactivation rate constant
$k_d^*$	second-order deactivation rate constant
$k_f$	forward rate constant for reversible monomer coordination or $\beta$ -agostic interaction; thermal conductivity
$k_{fL}$	effective thermal diffusivity in the macroparticle
$k_{fp}$	thermal conductivity of the polymer layer around the catalyst fragment in the microparticle
$k_{iH}$	rate constant for initiation of metal hydride active sites
$k_p$	propagation rate constant
$k'_p$	apparent propagation rate constant, Eq. (5.115)
$\hat{k}_p$	pseudo-propagation rate constant
$\tilde{k}_p$	apparent propagation rate constant
$k_{pi}$	propagation rate constant for monomer type $i$ (Bernoullian model)
$k_{pij}$	propagation rate constant for chain terminated in monomer type $i$ coordinating with monomer type $j$ (terminal model)
$k_{pijk}$	propagation rate constant for chain terminated in monomer types $i$ and $j$ coordinating with monomer type $k$ (penultimate model)
$k_{pm}$	propagation rate constant for meso insertion (propylene)
$k_{pr}$	propagation rate constant for racemic insertion (propylene)
$k_r$	reverse rate constant for reversible monomer coordination or $\beta$ -agostic interaction
$k_{tAl}$	rate constant for transfer to cocatalyst
$k_{t\beta}$	rate constant for $\beta$ -hydride elimination
$k_{tH}$	rate constant for transfer to hydrogen
$k_{tM}$	rate constant for transfer to monomer
$K$	Mark–Houwink equation constant, Eq. (2.7)
$K_a$	initiation frequency, $k_a[Al]$
$K_{eq}$	equilibrium constant for dormant sites, Eq. (5.67)
$K_{g-1}, K_{g-1}^*$	gas–liquid partition coefficients, Eq. (5.113)

$K_{g-s}$ , $K_{g-s}^*$ , $K'_{g-s}$	gas–solid partition coefficients, Eqs (5.111) and (5.114)
$K_{l-s}$	liquid–solid partition coefficient, Eq. (5.112)
$K_H$	Henry law constant
$K_T$	lumped chain-transfer constant, Eq. (5.74)
$K_T^1$	lumped chain-transfer constant, Eq. (5.70)
$K_T^H$	lumped chain-transfer constant, Eq. (5.71)
$m_i$	mass fraction of polymer made on site type $i$
$m_k$	mass fraction of chains with $k$ long-chain branches
$m_p$	mass of polymer, polymer yield
$\dot{m}_{\text{vap}}$	vaporization rate
$m_w$	molecular weight of repeating unit; in the case of copolymers, the average molecular weight of the repeating units
$M$	molecular weight
$M$	monomer
$M_C$	molar mass of catalyst
$M_n$	number average molecular weight
$M_v$	viscosity average molecular weight
$M_w$	weight average molecular weight
$MW$	polymer molecular weight
$n$	number of long-chain branches per chain; number of active site types
$n_c(v)$	polymer particle size distribution
$n_{C_0}$ , $n_{C_0}$	number of moles of catalyst
$n_{\text{LCB}}$	average number of long-chain branches in a polymer sample
$n_M$	number of moles of monomer
$n_w$	weight average number of long-chain branches per chain
$N_A$	Avogadro number
$N_i$	flux of species $i$
$N_s$	number of macroparticles per unit volume of the reactor
$Nu$	Nusselt number
$P_A$ , $P_B$	probability of propagation of monomers A and B, respectively
$P_H^*$	metal hydride active site
$P_M$	partial pressure of monomer
$P_p$	propagation probability
$P_r$	living chain with length $r$
$P_r^i$	living polymer chain with length $r$ terminated in monomer type $i$ (A or B for binary copolymers) or 1-2 or 2-1 insertions for polypropylene
$P_{r,i}^*$	living polymer chain of length $r$ having $i$ long-chain branches
$\tilde{P}_1$	dormant site due to $\beta$ -agostic interaction, Table 5.5
$\tilde{P}_r$	dormant site for Ni-diimine catalysts, Table 5.8
$P_t$	termination probability
$PDI$	polydispersity index

$\overline{\text{PDI}}$	polydispersity index for chains containing long-chain branches
$\text{PDI}_{F_A}$	polydispersity index as a function of copolymer composition
$\text{PDI}_k$	polydispersity index for chain with $k$ long-chain branches, Eq. (6.106)
Pr	Prandtl number
$\dot{Q}$	heat generation rate
$r$	polymer chain length
$r_i$	comonomer reactivity ratio
$r_L$	radial position in macroparticle (multigrain model)
$r_n$	number average chain length
$\bar{r}_n$	number average chain length for chains containing long-chain branches, Eq. (6.107)
$\tilde{r}_n$	number average molecular weight that would result in the absence of long-chain branch formation reactions, see footnote 13 in Chapter 6
$r_{nF_A}$	number average chain length as a function of copolymer composition
$r_{nk}$	number average chain length for chains with $k$ long-chain branches, Eq. (6.103)
$r_s$	radial position in the microparticle (multigrain model)
$r_w$	weight average chain length
$\bar{r}_w$	weight average chain length for chains containing long-chain branches, Eq. (6.108)
$r_{wF_A}$	weight average chain length as a function of copolymer composition
$r_{wk}$	weight average chain length for chains with $k$ long-chain branches, Eq. (6.104)
$r_{zk}$	z-average chain length for chains with $k$ long-chain branches, Eq. (6.105)
$\overline{r_0^2}$	root-mean-square end-to-end distance of a polymer chain
$R$	gas constant
$R_c$	catalyst fragment radius (multigrain model)
$R_i$	reaction rate of species $i$
$\langle R_g^2 \rangle_b$	squared radius of gyration of branched chains
$\langle R_g^2 \rangle_l$	squared radius of gyration of linear chains
$R_L$	macroparticle radius (multigrain model)
$R_p$	polymerization rate
$\overline{R_p}$	average polymerization rate per unit volume of the reactor
$\overline{R_p}$	average polymerization rate per polymer particle
$R_s$	microparticle radius (multigrain model)
$R_t$	chain-transfer rate
Re	Reynolds number
$\Delta S$	entropy change
Sc	Schmidt number
Sh	Sherwood number
$t$	time

$\bar{t}$	average reactor residence time
$t_R$	reactor residence time
$t_{\frac{1}{2}}$	catalyst half-time
$T$	temperature
$T_c$	crystallization temperature
$T_{i0}$	reactor inlet temperature
$T_m$	melting temperature
$T_m^0$	melting temperature of an infinitely long polyethylene chain
$T_S$	temperature in the microparticle
$T_w$	reactor coolant temperature
$U$	global heat-transfer coefficient
$V$	Monte Carlo control volume; reactor volume
$V_e$	elution volume
$V_i$	interstitial volume
$V_p$	pore volume
$V_R$	reactor volume
$w_{\log r}$	weight Flory chain length distribution, log scale, Eq. (6.24)
$w_r$	weight Flory chain length distribution, Eq. (6.17)
$\bar{w}_r$	cumulative weight Flory chain length distribution
$\overline{w}_r$	overall weight chain length distribution for chain having long-chain branches, Eq. (6.102)
$w_{r,FA}$	Stockmayer bivariate distribution, Eq. (6.60)
$w_{rk}$	weight chain length distribution for chains with $k$ long-chain branches per chain, Eq. (6.87)
$w_{\log rk}$	weight chain length distribution for chains with $k$ long-chain branches per chain, log scale, Eq. (6.89)
$w_{\log MW,FA}$	Stockmayer bivariate distribution, log scale, Eq. (6.61)
$w_{r,FA}^k$	trivariate distribution of chain length, chemical composition, and long-chain branching, Eq. (6.117)
$w_{ry}$	Stockmayer bivariate distribution, Eq. (6.56)
$w_{\log MW}$	weight Flory molecular weight distribution, log scale, Eq. (6.32)
$w_{MW}$	weight Flory molecular weight distribution, Eq. (6.30)
$W_C$	mass of catalyst
$x_c$	mass fraction of catalyst in a supported catalyst
$x_i$	molar fraction of comonomer $i$ in the copolymer
$\gamma$	deviation from average comonomer molar fraction in the copolymer, Eq. (6.57)
$\gamma_k$	molar fraction of chains with $k$ long-chain branches
$Y_i$	$i$ th moment of living polymer
$[Y_0]$	total concentration of active sites or living polymer chains
$Z$	compressibility factor

**Greek Letters**

$\alpha$	polymer chain hydrodynamic volume constant, Eq. (2.4); long-chain branching parameter, Eq. (6.93)
$\beta, \hat{\beta}$	Stockmayer bivariate distribution parameters, Eqs. (6.58) and (6.62), respectively
$\varepsilon$	exponent relating $g'$ to $g$ , Eq. (2.19); macroparticle void fraction
$\eta$	catalyst site efficiency, Eq. (5.45)
$[\eta]$	intrinsic viscosity
$\phi$	fraction of active sites with growing polymer chains, Eq. (5.55)
$\phi_i$	fraction of living chains terminated in monomer type $i$
$\phi_k$	Catalan numbers, Eq. (6.95)
$\Phi$	polymer chain hydrodynamic volume constant defined in Eq. (2.4)
$\kappa$	size exclusion partition coefficient
$\lambda, \lambda_n$	number of long-chain branches per 1000 C atoms
$\mu$	long-chain branching parameter, Eq. (6.94); viscosity
$\frac{\rho_C}{\rho C_p}$	support (catalyst) density average value of the heat capacity per unit volume of the macroparticle
$\tau$	Flory most probable chain length distribution parameter; ratio of all chain-transfer rates to the propagation rate, Eq. (6.29); $g$ macroparticle tortuosity
$\hat{\tau}$	Flory most probable molecular weight distribution parameter, Eq. (6.31)
$\tau_B$	chain length distribution parameter for polymers containing long-chain branches, Eq. (6.90)
$\tau_d$	characteristic diffusion time in the macroparticle

**Superscripts and Subscripts**

$\wedge$	pseudokinetic constant
$-$	average
12, 21	1-2 or 2-1 propylene insertions
A, B	monomer types
bulk	bulk conditions
C	catalyst, monomer type C in the case of terpolymerization
l	liquid phase
M	monomer
MC	Monte Carlo simulation rates and constants
P	polymer
s	solid polymer phase